Forest planning and productivity-risk trade-off through the Markowitz mean-variance model

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Abstract

By means of the Markowitz mean-value (M-V) portfolio model, we study the forest planning so as to to make a comparative assessment between productivity and risk. By weighting the forest productivity with factors of future climate change effects, we compute the optimal tree species mixes, within reach of forest managers, in ninety French administrative departments. Considering three different productivity measures (wood production, carbon sequestration and economic valorization) and their respective variances, we find that: a) the empirical allocation lies between the optimizations of wood production and economic valorization; b) forest managers prefer low variance to high productivity, i.e. their revealed risk aversion is high; and c) unlike maximizing wood productivity or carbon sequestration, which leads to similar portfolios, maximizing the economic value of wood production decreases both the levels of wood production and carbon sequestration. Under high risk aversion, the economic valorization would lead to a high species specialization, which is very unlikely in reality. In all considered scenarios, the objectives set out in the Kyoto Protocol would be attained, which puts into question its relevance in terms of additionality.

Keywords: bioeconomics, forest planning, mean-variance model, mixed-species forests, climate change

JEL Classification: G17, Q2, Q54

Contents

1 Introduction

Due to climate variations, as well as biotic and abiotic disturbances, the services provided by forest ecosystems are characterized by their strong fluctuations. Furthermore, climate change is expected to alter the provision of these services in a way that is far from being fully understood (Millar et al., 2007). On one side, the increase of the CO_2 atmospheric concentration may lead to the *carbon fertilization effect*, according to which the growth rate of tree species should increase (Soulé & Knapp, 2006; Knapp et al., 2001). On the other side, climate change may accentuate the risk of tree mortality (Allen et al., 2010; Lindner et al., 2010; Dale et al., 2000).

The objective we have set is to describe a methodology that, selecting a particular mix of tree species, could help to shape the forest ecosystems such that the provision of services is both maximized and resilient to external shocks. For instance, the optimal mix of tree species could lower the risk of seeing the level of forest services deteriorated in the face of climate change.

As regards the forest management, we consider the preferences of forest managers to lie within a continuum between risk aversion and risk neutrality. Put differently, when forest resources are treated as investments that could generate a level of expected utility, their managers would not invest in a combination of tree species – a silvicultural portfolio – if a more favorable portfolio, with different expected return and risk, was achievable. In that sense, the forest manager is considered to be rational, for he or she will be looking for a portfolio that generates the greatest expected utility (Kumar et al., 2014).

The trade-off between the expected return of a portfolio of assets and its combined variance has initially been discussed by Markowitz (Markowitz, 1952) through his mean-variance (M-V) model. The latter then extensively applied, as an arbitrage tool, to numerous economic sectors, including forestry (Pasalodos-Tato et al., 2013). In such a model, a specific weighted combination of assets, such as tree species, is selected in order to minimize the portfolio variance subjected to a given target return or, equivalently, so as to maximize the expected return given an acceptable level of variance.

When applied to forestry, the portfolio analysis has been employed from the point of view of the forest managers when they behave as investors, where the investments in timberland are balanced against other types of investments (e.g. stocks or bonds) in order to maximize the portfolio financial return (Thomson, 1991; Wan et al., 2015). Alternatively, the M-V model has been employed as a decision aid tool to deal with risk and uncertainty, with a portfolio of tree species analyzed either at the stand level (Knoke et al., 2008; Knoke, 2008; Roessiger et al., 2011), the management level (Knoke et al., 2005; Neuner et al., 2013), or the regional level (Brunette et al., 2014).

Most of the studies aforementioned are based on the historical distribution analysis, choosing a distribution, fitting its parameters to the observed one, and using Monte Carlo simulations to produce random series. Contrariwise, this paper follows the work by Brunette et al. (2014) and directly uses historical data issued from the French National Forest Inventory (IGN). The main advantage of such method is that it requires less data and needs fewer assumptions on the distribution choice and parameters. However, by using historical distributions, we are faced with potential overfitting issues when the results are projected toward other time periods, and may also underestimate the extreme events with fat tailed distributions.

While our model considers three objectives that can be assigned to forest ecosystems (Wood Production – WP, Carbon Sequestration – CS, Economic Value – EV), the optimization has been conducted using the species and department specific historical observations of tree growth.

The literature in forest ecology usually states that, for individual trees or tree populations, a declining growth level, as compared with the the species potential, presents a high mortality risk, for the indicator reflects the tree vigor and is indicative of its survival likelihood (Buchman et al., 1983; Bigler et al., 2004; Dobbertin, 2005). Moreover, many recent works suggest that a high variance in tree growth reflects a high risk of mortality (Ogle et al., 2000; Suarez et al., 2004; McDowell et al., 2010; Heres et al., 2012). Thereby, the environmental stress produces an exaggerated variation of tree-rings, such that greater sensitivity to stress comes down to greater mortality (Hogg et al., 2005; Linares & Camarero, 2012). The amplitude of variation of productivity is thus considered to be a measure of risk (Tilman et al., 1997; Andreu et al., 2007; Slimani et al., 2014).

This paper extends the Markowitz portfolio selection of Brunette et al. (2014): (a) for different levels of risk aversion exhibited by forest managers; (b) for different climate change scenarios during the optimal allocation; (c) to different maximization objectives, such that WP is compared with CS and EV.

As the portfolio expected output is computed from the historical data, the implicit assumption is that the expected productivity of species would be equivalent to the ones currently observed. However, this invariability assumption is mitigated by the fact that the portfolio simulations are conducted at a relatively small scale, that is, the French administrative departments.

Through simulations, our model yields the following results: a) the empirical allocation stands between the optimizations of wood production and economic valorization; b) forest managers prefer low variance to high productivity, i.e. their revealed risk aversion is high; and c) unlike maximizing wood productivity or carbon sequestration, which lead to similar portfolios, maximizing the economic value of wood production decreases both the levels of wood production and carbon sequestration. Under high risk aversion, considering the economic value, rather than the wood productivity, would lead to a high specialization in tree species. This is neither likely nor desirable due to the risk which would result from low diversification, not to mention the change of scenery. Considering either scenario, the objectives set out in the Kyoto Protocol would be attained.

After this starting section, the methodology we have used is presented in Section 2. Section 3 is devoted to illustrating simulation examples. Section 4 concludes.

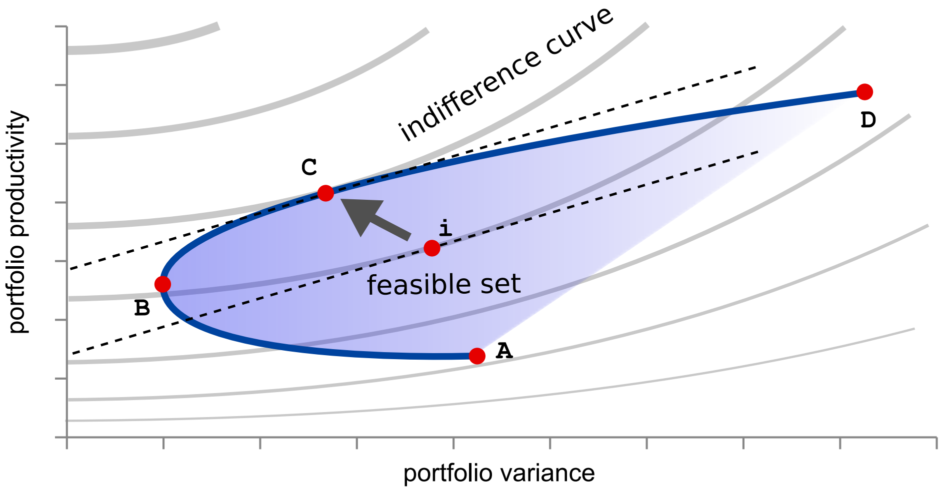
2 Methodology

All the possible combinations of species define the feasible portfolio set. Fig. 1 portrays the productivity-variance space, where the set (such as point `i`) in enclosed by the blue curve and the upper segment of the parabola (`B-D` segment) represents the *efficient frontier* (EF), that is, all the optimal allocations achievable by the decision maker. Thereby, no risk can be lowered at the expense of the productivity level and no productivity can be enhanced without increasing the risk. Productivity itself can be defined in different terms. For instance, Section 2.4 takes physical growth of timber production, carbon sequestration and soil expectation value as different expressions of productivity. The indifference curves between productivity and variance (the thicker the line, the greater the utility) are drawn in grey.

Under the standard neoclassical assumption of concave utility functions with respect to a single good, indifference curves in the productivity-variance space become convex (Pennacchi 2007, ch.2). EF being concave (Merton, 1972), this guarantees the presence of a single optimal point that maximizes the agent's utility.

At this optimal point (`C`), the tangent to the indifference curve is equal to the tangent to the efficient frontier, which equation is defined as $p = \alpha \times v + \beta$, where α is the linear risk aversion coefficient dp/dv – defined as the productivity (p) that agents require to accept more variance (v) –, and p and v refer to the overall expected productivity and variance of the portfolio. The parameter β is the intercept with the ordinates of the tangent, where variance is equal to zero. As indifference curves do not intersect, maximizing β corresponds to maximizing the utility of the certainty equivalent.

Figure 1: Graphical representation of the portfolio allocation



Point A is the one with the lowest portfolio combined productivity. Instead, B represents the point at which the portfolio variance is at its lowest, while the tangent to the efficient frontier has the highest value of slope. Choosing B as optimal point would imply an infinite risk aversion ($\alpha \to \infty$). Agents with α-level of risk aversion are expected to choose point C, at which the indifference curve and its tangent intersect the efficient frontier. Mathematically, it boils down to solving the following quadratic problem, where α remains exogenous and must be pre-defined:

$$\begin{aligned} \max_{x_i,\beta} \quad & \beta \\ s.t. \quad & \\ x_i &\geqslant 0 \qquad \forall i \\ \textstyle\sum_i x_i &= 1 \\ \textstyle\sum_i x_i y_i &= \alpha \textstyle\sum_i \sum_j x_i x_j \sigma_{i,j} + \beta \end{aligned} \tag{1}$$

where x_i is the share of asset i, y_i is its productivity and $\sigma_{i,j}$ is the covariance between assets i and j. In this way, $\sum_i x_i y_i$ is the overall portfolio productivity and $\sum_i \sum_j x_i x_j \sigma_{i,j}$ its corresponding variance. By substitution, Eq. 1 becomes:

$$\begin{aligned} \min_{x_i} \quad & \alpha \textstyle\sum_i \sum_j x_i x_j \sigma_{i,j} - \sum_i x_i y_i \\ s.t. \quad & \\ x_i &\geqslant 0 \quad \forall i \\ \textstyle\sum_i x_i &= 1 \end{aligned} \tag{2}$$

Finally, point D (where $\alpha = 0$) is the highest portfolio productivity attainable by the decision maker. Despite its performance, it is more a degenerated solution where only the most productive species would remain.

Proposition 1 *The portfolio allocation problem of Eq. 2 is a strictly convex minimization problem.*

Proof. The objective function of Eq. 2 is a sum of linear functions and quadratic terms. Provided that the quadratic terms, which only arise when $i = j$ and $\sigma_{i,j} \equiv \sigma_i^2$, are always positive, they are strictly convex and so is the function (Chiang & Kevin, 2005). ■

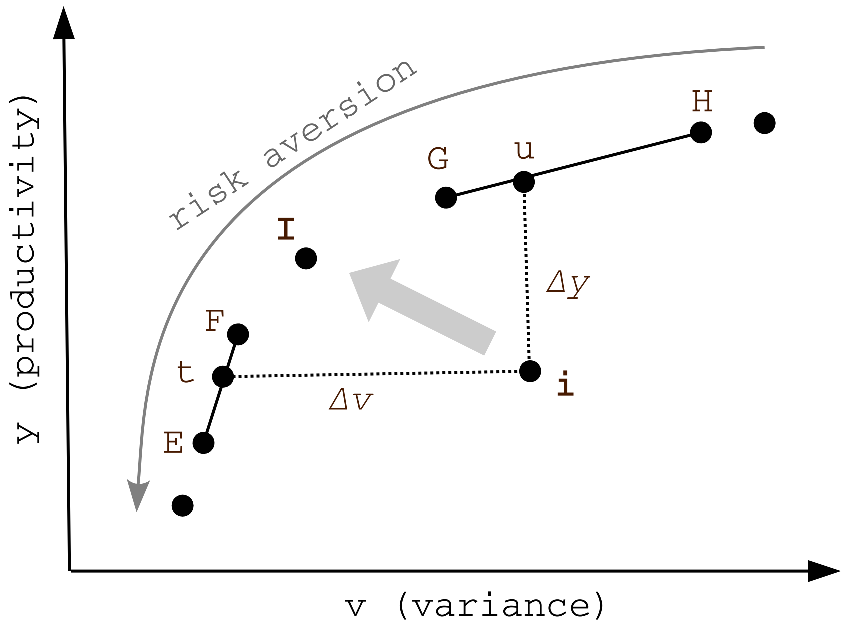
The bounds being linear, we employ QuadProg++ (Di Gaspero, 2007) in order to numerically solve the problem in 2. By means of an active-set dual method, the former is a library which implements the algorithm of Goldfarb & Idnani (1983) for the (convex) quadratic programming problems.

2.1 Empirical risk aversion

Through the use of the M-V model, we measure the distance between the current (supposedly inefficient) allocation of tree species and an optimized portfolio. As the empirical point `i` is contained in the space bounded by the efficient frontier, we aim at revealing its corresponding risk aversion coefficient (α_i) and compare the portfolio at point `i` with the point belonging to EF characterized by the same risk aversion coefficient. To do so, we use a simple linear interpolation such as sketched in Fig. 2 .

Assuming the sole productivity improvement, which corresponds to moving the y-axis coordinate toward the frontier – from `i` to `u` in the Figure –, would imply a change in risk aversion; instead, a risk-constant agent would use part of that gain in efficiency to reduce the variance, hence moving from point `i` to point `I`.

Figure 2: Representation of the interpolation to retrieve the risk aversion



Given a predetermined sets of risk aversion coefficients, we can run the optimization according to eq. 2 to retrieve the corresponding optimal points on EF (points `E`, `F`, `G` and `H`, among others). We know the risk aversion coefficient (as the exogenous parameter of the optimization), as well as the productivity and variance (obtained after the optimal portfolio is computed) at each of these points. Given the observed allocation point `i`, points `G` and `H` are part of the computed points, which variances are respectively lower and higher than those observed at `i`. Similarly, points `E` and `F` are the points which productivities are below and above those at `i`.

By linear interpolation, we first find the risk aversion coefficients of points `t` and `u` (respectively α_t and α_u), and then weigh them using distances $\triangle v$ and $\triangle y$ to retrieve the risk aversion of `I` (which was not included in the initial set of risk aversions):

$$\begin{aligned}
\alpha_t &= \frac{\alpha_F-\alpha_E}{y_F-y_E} \times (y_i - y_E) + \alpha_E \\
\alpha_u &= \frac{\alpha_H-\alpha_G}{v_H-v_G} \times (v_i - v_G) + \alpha_G \\
\triangle v &= v_i - \left(\frac{(v_F-v_E)(y_i-y_E)}{(y_F-y_E)} + v_E\right) \\
\triangle y &= \left(\frac{(y_H-y_G)(v_H-v_G)}{(v_H-v_G)} + y_G\right) - y_i \\
\alpha_i &= \alpha_t \times \frac{\triangle y}{\triangle v+\triangle y} + \alpha_u \times \frac{\triangle v}{\triangle v+\triangle y}
\end{aligned} \tag{3}$$

Once we found the risk aversion coefficient through the above interpolation, in a second step we add it to the pre-established list of coefficients, so that we could re-run the simulations and find the optimal portfolio at point `I`, the equivalent to point `C` in Figure 6. The results relative to the *intermediate* risk aversion in Tables 3 and 4 and the white dashed line in Figures 5 and 7 refer to this point, as computed from the data interpolation at national level.

2.2 Productivity data

The data we used, on eleven tree species present in France, comprises the 1978-2009 time length. The database, coming from the French National Forest Inventory (IFN[1]), included the volume growth, as well as the area occupied per species, per department and per year. Productivity is then simply derived as the volume growth divided by the occupied area.

The data happens to be relatively sparse for two reasons. First, individual tree species are often present in a subset of departments. Second, between 1978 and 2006, the annual inventory concerned few departments, with a time gap of 10 to 12 years between two inventories in the same department. In 2004, the sampling method changed (IFN, 2004), such that all departments could be simultaneously inventoried. This method has become operative in 2007.

Table 1 shows the number of departments in which the species is present and the number of years during which it has been identified. For example, *Abies alba* Mill appeared 5 times in 35 departments but only 3 times in other 16 departments. Each aggregated data comes from an extensive number of plots' observations per department, species and year (average: 93; median: 50). From this data, we obtain the (measured) current annual increments. To treat them as the average annual forest increment, we need to assume that the National Inventory sampling is consistent across the diameter classes.

In order to build the covariance matrix $\sigma_{i,j}$ from eq. 2, we decided to consider species, in each given department, for which we had a minimum of 4 observations well spread among the observation period. To fill the data gaps, we used the standard linear interpolation. After removing the species with limited observations, fewer of them were available to compute the optimal portfolios, such as specified in Table 2.

2.3 Climate change multipliers

Climate change multipliers describe the effects of a climate scenario on the variation of the average growth of tree species. By means of a statistical procedure described below, they have

[1]In 2012, the French National Geographic Institute and the French National Forest Inventory have merged into the French National Institute of Geographic and Forest Information (IGN)

Table 1: Presence of forest species in inventories at the department level

	Number of years with observations					
	1	2	3	4	5	Total
Abies alba Mill.	2	3	16	1	35	57
Fagus sylvatica L.		2	11	1	64	78
Larix decidua Mill.	4	2	16		5	27
Picea abies L.	1	5	6	2	47	61
Pinus pinaster Aiton	1	4	8	1	29	43
Pinus sylvestris L.	1	4	13	3	58	79
Pseudotsuga menziesii Franco	3	1	23	1	43	71
Quercus ilex L.	1		5	1	12	19
Quercus petraea Liebl.		1	7	3	73	84
Quercus pubescens Willd.	1	4	22	1	30	58
Quercus robur L.		2	4	1	74	81
Total	14	28	131	15	470	658

Table 2: Frequency of the species numerosity in the portfolio per department

N species accounted	N dep. orig	N dep. after filter
2	0	3
3	0	8
4	5	15
5	6	21
6	18	20
7	18	17
8	22	4
9	14	1
10	5	1
11	2	0

Dimension of the portfolios (number of species accounted for), number of department concerned in the original IGN data and number of departments after filtering-out species with limited data.

been computed for 7 different species[2] by J.D Bontemps and P. Mérian of the Laboratory for the Study of Forest-Wood Resources (LERFoB)[3].

The empirical growth rates – from the IGN data on radial growth obtained by core drilling of the stems – have been correlated to both edaphic and climatic data – the SAFRAN data over the period 1958-2010 – on a high-resolution scale (8 km resolution grid) using a generalized additive model (GAM). The growth rates have then been projected using the CERFACS future climate scenarios.[4] The projections covered the years ranging from 2000 to 2100 for the IPCC[5] scenarios a1b, a2 and b2, which are issued from the ARPEGE-Climate model. The projected growth rates have finally been downscaled at a regional level resolution and converted to multipliers. Tree productivity in the future (by each species, department and year) is then obtained, conditionally to the specific IPCC scenario, multiplying the last year of the observed productivity (that is, the reference period) by the multiplier. It is important to notice that, in this particular study, the only considered climate change effect is the variation in the trees' growth patterns. Other

[2]When a multiplier of a species in Table 4 was not available, we employed the existing multiplier of the species with the closest ecological needs.

[3]INRA–AgroParisTech UMR 1092, Laboratoire d'Etude des Ressources Forêt-Bois (LERFoB), 14 rue Girardet, 54000 Nancy, France

[4]Centre Européen de Recherche et de Formation Avancée en Calcul Scientifique

[5]Intergovernmental Panel on Climate Change

effects, both physical (e.g. changes in the mortality rates) and economic (e.g. price volatility due to changes in demand), which can also play an important role, have not been considered.

2.4 Three objectives

Historically, a key issue in forest management has been the choice of optimal rotation length. The latter has been covered by the literature from the early eighteen century to nowadays, often considering the role of risk and other new objectives, in primis carbon sequestration, as new research questions (see the literature reviews of Newman 2002; Yousefpour et al. 2012).

When using the optimal choice of species as a free variable, instead of the optimal rotation length, maximizing a portfolio of species for its (undiscounted) productivity (mean annual increment) is equivalent to maximizing the Maximum Sustained Yield (MSY). On the contrary, using the soil expectation value would correspond to the "Forest Rent" doctrine popularized by Faustmann (1849).

While the forest rent approach is economically more efficient (Hyytiäinen & Tahvonen, 2003), the current objective followed by forest practitioners remains unclear. Möhring (2001) suggests that MSY remains the predominant management guideline in the Central European forests. This may be particularly true for public forest agents, that need to guarantee both the supply of wood to the downstream industries and the sustainability of the forest resources. Nautiyal (1988), as cited in Saastamoinen & Matero (2004), associates the popularity of MSY to its simplicity.

For this reason, we propose an extension of Brunette et al. (2014). While the authors report on optimal portfolios considering the sole physical wood production (WP), we include carbon sequestration (CS) and economic valorization (EV, that is, the maximization of the forest rent), to see which approach would lead to the portfolios similar to the current ones.

2.4.1 Carbon sequestration

While undoubtedly a temporary solution, carbon sequestration in carbon pools with long-term turnover (e.g. forests) is a relatively cheap and quick form of Carbon Dioxide Removal Method. We know that the latter can help reducing the cumulative impact of higher temperature (Ciais & Sabine, 2013; Smith & Bustamante, 2014). We thus introduced a portfolio objective in which the optimal allocation would maximize the carbon sequestered in the forest stands. We multiplied the wood productivity by a CO_2 factor $F_s^{CO_2}$ for each species s:

$$F_s^{CO_2} = wd_s \times cc_{gs} \times expf_s^b \times expf_s^r \times \frac{44}{12} \tag{4}$$

where wd_s is the wood density by species defined over the oven dry mass over the fresh volume (Chave et al., 2009; Zanne et al., 2009), cc_{gs} is the carbon content by group of species gs (hardwood/softwood) (Lamlom & Savidge, 2003) and $expf_s^b \times expf_s^r$ are respectively the branch and roots expansion factors (Loustau, 2004).

As the carbon sequestration is realized through the whole forest growth and not only at the rotation end, we maintain the undiscounted wood productivity as a base.

The output is a sequestration productivity per hectare and per year. To fit the standard terminology of international negotiations framework on greenhouses gases, we refer to the CO_2 equivalent throughout this paper, even if only CO_2 is considered in our case.

Let us specify that we do not attempt to discuss the trade-off between forest carbon sequestration, carbon substitution in the harvested wood products (HWP) or the substitution effect (with regard to fossil fuels) from timber (see Lecocq et al. 2011; Kindermann et al. 2013; Miner et al. 2014; Smyth et al. 2014; Ter-Mikaelian et al. 2015 for explicit discussions on this topic). While we simply account for the sequestered carbon in commercial stands, we do not give reason for its usage inside or outside the forest areas. Accordingly, when comes to HWP, we do not consider the lifetime peculiarity (before oxidation) which is species specific, and thus the "optimal carbon sequestration" allocations could be biased toward species with relative short-term HWP, like the pines.

2.4.2 Economic value

In a similar way, we optimized the forest portfolio for the economic valorization objective, that is, the soil value obtained from the harvesting discounted value at the rotation end. As we do not have a systematic dataset on costs per tree species at the required geographical level, we implicitly assumed equal regeneration and operational costs per tree species.

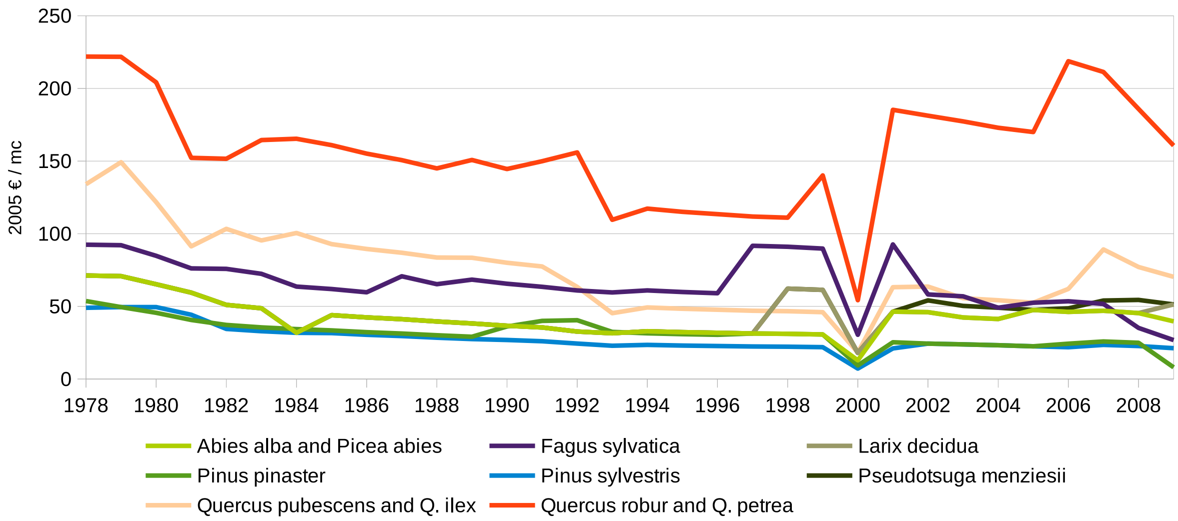
The original wood productivity per species in a given year ($y_{i,t}$, that in the climate change simulations already includes the multipliers) has been transformed into production ($Y_{i,t}$) multiplied by the species typical rotation length (T_i) and then into value by multiplying the production by the roadside timber prices ($p_{i,t}$, in constant terms). The cash-flows have been discounted at a discount rate (dr) of 3% commonly used in forestry (Möhring, 2001). In order to take different rotation lengths into account, the discounted value has finally been transformed into the soil expectation value ($sev_{i,t}$). The overall transformation is given by Equation 5, where out-of-sample prices, used in climate change scenarios (after 2009), are frozen to the last observed value.

$$sev_{i,t} = \frac{y_{i,t} * p_{i,t} * T_i}{(1 + dr)^{T_i} - 1} \quad (5)$$

In the EV maximization objectives, $sev_{i,t}$ are both used as "productivity" and as means to build the economic value covariance matrix. As for the other two objectives, the covariance matrix has been computed for the in-sample period. Together with the modifying productivities, it was then applied for the climate change scenarios.

Fig. 3 shows the evolution of prices during the study period, with large heterogeneities in the absolute values and their variances. From what we observe, the lower the price, the lower the variance.

Figure 3: Evolution of main wood prices per cubic meter and per species in France



The price data was collected by the French newspaper *La forêt privée*,[6] diffused twice a month, from 1958 onwards. A detailed description can be found in (Chevalier et al., 2011). We

[6] http://www.laforetprivee.com

used the annual mean of maximum and minimum prices, issued from the bids recorded during the auction sales in the considered French departments. We matched the tree species and the aforementionned available prices. For each species, only the prices of the highest wood quality have been taken into account (first choice and straight logs/boles).

3 Simulations

Given the productivity data detailed in Section 2.2, and the multipliers explained in Section 2.4, we implemented a simple computer program so as to find the allocation of tree species which maximizes wood production at a minimum variance.

The efficient frontier was computationally obtained (point by point) optimising the problem of Equation 2 using a pre-defined set of 14 risk aversion coefficients, ranging from 0 (point `D` in Fig. 1) to 10,000. We then optimized the portfolio problem for point `B` by selecting the species which best performed in terms of minimum variance. In order to account for 90 administrative departments, 14 risk aversion coefficients, 3 climate change scenarios, 3 different objectives, 5 time spots, and two discount rates (0 and 0.03) a total of 117,180 simulations had to be run.

The computational constraint of maintaining the optimization problem in its quadratic form prevented us from using the standard deviation or a more elaborated measure of risk, such as the value at risk (VaR) or the conditional value at risk (CVaR) (Wan et al., 2015). In particular, using variance as a risk measure implies that α is defined over the metric used to measure productivity. In order to maintain the meaning of α consistent, we normalized the different measures of productivity from Section 2.4 before running the allocation problem.

While the expected productivity of the forest species has been computed from 2010 to 2100, in twenty-years steps, using the climate change multipliers described in Section 2.3, the covariance matrix has been maintained fixed.

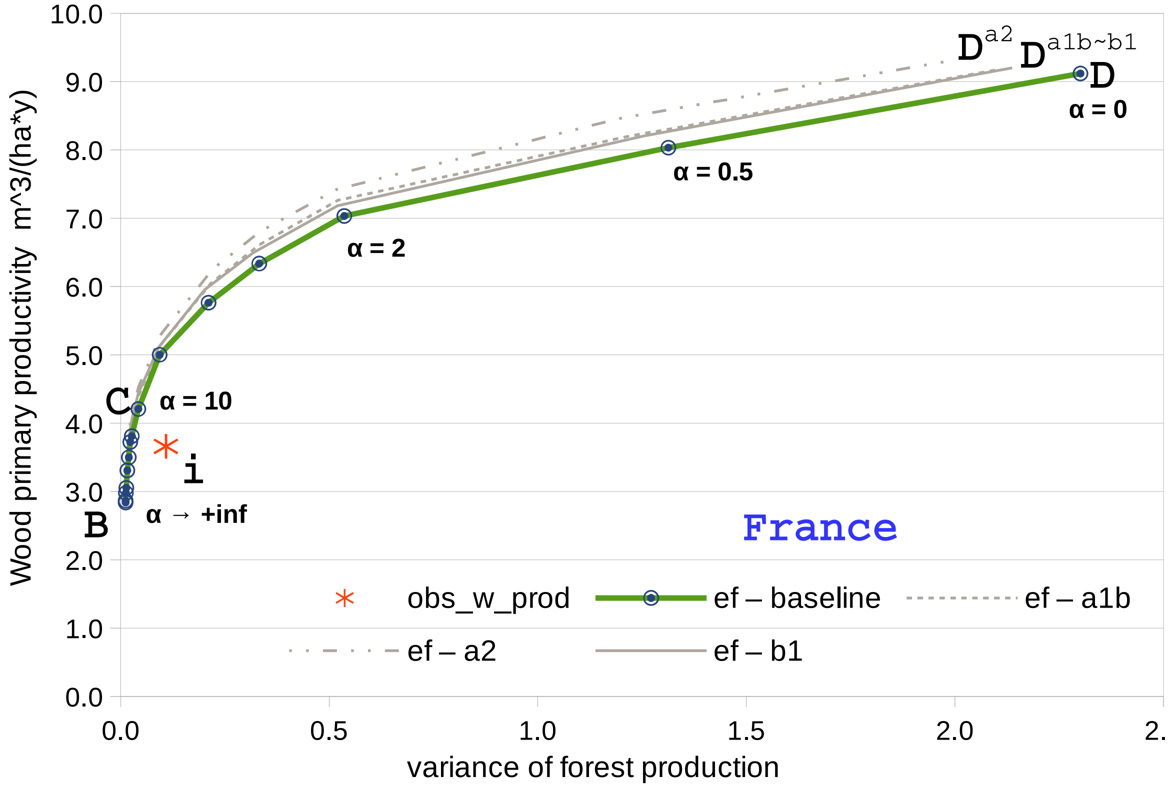
Fig. 4 displays the efficient frontier (EF) calculated from the current productivities (green curve), as well as the allocations relative to the IPCC scenarios (grey curves). We also observe a red star, which represents the actual French allocation. It can be discerned that, at the national level, the empirical forest allocation is close to the efficient frontier. Nevertheless, the high risk aversion that we reveal places the management of French forests at the low parabola coordinates, where the portfolio productivity is not at its highest.

Optimizing the current portfolio for wood production (moving from point `i` to point `C` in Fig. 1), while keeping the same risk aversion, would lead to the respective increases in wood production and sequestration of 3.2 $M\ m^3\ y^{-1}$ and 3.2 $Mt\ \mathrm{CO_2}eq\ y^{-1}$ (Table 3). Regarding the economic value, optimizing the wood production, without targeting its economic valorization, would lead to a a value fall of 416 $M€\ y^{-1}$. Once we apply the interpolation method described in Section 2.1, we fall on a national-level risk aversion coefficient equal to 70.58 $m^3\ ha^{-1}\ y^{-1}$, that is, on average, forest managers would require an increase of 7.7 $m^3\ ha^{-1}\ y^{-1}$ to compensate for a doubling of the current variance in wood production.

Fig. 4 also reports the way EF would be modified if a specific climate change scenario, defined in Section 2.3, occurred. Given that the covariance matrix is fixed, the EF transposition results from changes in the expected productivity. Thereby, for any given scenario, the respective EF curves coincide at the minimum variance coordinates and diverge as α decreases. Put differently, we find that the climate change scenarios positively affect the frontier, for the productivities increase at any given level of risk. In detail, the a2 scenario performs the best. We recall however that the only climate change effect considered in these simulations is the trees' growth rate. In this case, the risk aversion level of forest managers, that is, different points along EF, has greater impact on the forest production than climate change. Considering other effects may however lead to very different outcomes.

For all three objectives (Table 3), the proximity between point `C` and point `B` points out that, contrary to the risk neutral point (`D`), the effects of the intermediate risk aversion and full risk

Figure 4: Efficient frontier and actual allocation in France



Efficient frontiers (lines) and actual allocation (red star) for France. Baseline EF is 2009. Climate change scenarios are average values (2020-2100).

aversion are analogous.

Table 3: Wood production, carbon sequestration and economic valorization on different points of the efficient frontier (optimization with respect to the wood production)

Climate change scenarios	a1b	a2	b1	baseline
Wood production ($M\ m^3\ y^{-1}$)				
(currently observed: 55.4 $M\ m^3\ y^{-1}$; 3.52 $Mc\ ha^{-1}\ y^{-1}$)				
Full risk aversion	45.6	47.0	46.5	43.8
Intermediate risk aversion	60.2	62.8	61.1	58.6
Risk neutral	136.7	139.5	137.0	136.3
Carbon sequestration (Mt CO_2eq y^{-1})				
(currently observed: 78.8 Mt CO_2eq y^{-1}; 5.01 t CO_2eq $ha^{-1}\ y^{-1}$)				
Full risk aversion	69.4	71.8	71.0	66.3
Intermediate risk aversion	84.8	88.6	86.7	82.0
Risk neutrality	169.9	173.1	172.9	170.4
Economic valorization ($M€\ y^{-1}$)				
(currently observed: 3,789 $M€\ y^{-1}$; 240 € $ha^{-1}\ y^{-1}$)				
Full risk aversion	3,849	3,894	3,880	3,745
Intermediate risk aversion	3,637	3,734	3,687	3,373
Risk neutrality	5,312	5,278	5,408	5,426

Choosing a specific level of risk aversion significantly impacts the compositions in the optimal portfolios. Fig. 5 illustrates the relative species allocation under different assumptions on risk aversion. Three different patterns can be identified.

The first one is relative to the species that yield high portfolio productivity and risk: either because they show a high variance or because they are positively correlated with other portfolio species. These species (e.g. *Picea abies*, *Pinus pinaster* or *Pseudotsuga menziesii*) constitute an important part of the portfolio under risk neutrality.

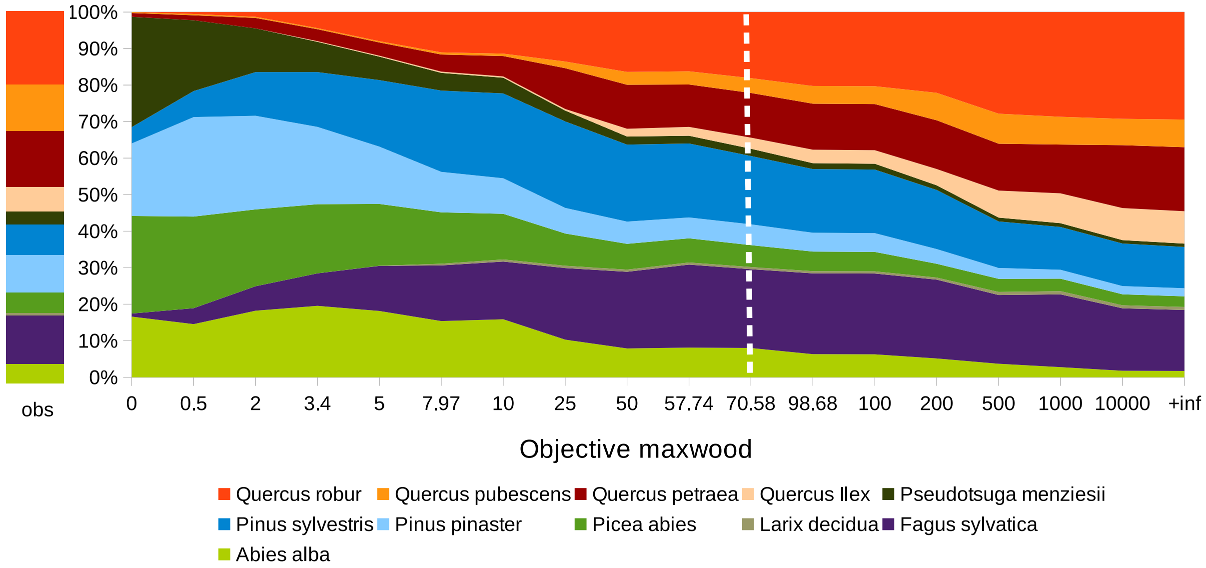
The second pattern includes the species (e.g. *Quercus robur*, *Quercus petrea*, *Quercus pubescent* and *Quercus ilex*) with specular characteristics: they bring stability to the portfolio to the detriment of its productivity. They tend to appear in greater proportions as the risk aversion increases.

The third pattern displays intermediate characteristics and arises under the intermediate risk aversion (e.g. *Pinus sylvastris* and *Fagus sylvatica*). Its coefficients yield the highest portfolio diversification.

At the current risk aversion level (white dashed line) the model suggests, compared with the actual distribution, an increased utilization of *Fagus sylvatica*, a partial substitution of *Pinus pinaster* with *Pinus sylvestris* and a general regression of *Quercus*, in particular *Q. pubescent*.

One of the critiques being leveled at the M-V model is that it looks at the past variance, such that the variance-covariance matrix is assumed to be constant. While point B reflects the most robust portfolio under the risk currently observed, the diversity encountered at the intermediate levels of risk aversion can also ensure an overall stability required for confronting climate change.

Figure 5: Species allocation by risk aversion



3.1 Regional differences

Although working with departmental data allow us to build the efficient frontier at the department level, we did not quantitatively measure the risk aversion coefficient or the distance to EF at constant risk aversion per department.[7]

[7]As explained in Sec. 2.1, this is a post-processing operation, so we only did it for the whole French territory; it would be too time-consuming to do it for the 90 administrative departments. Nonetheless,

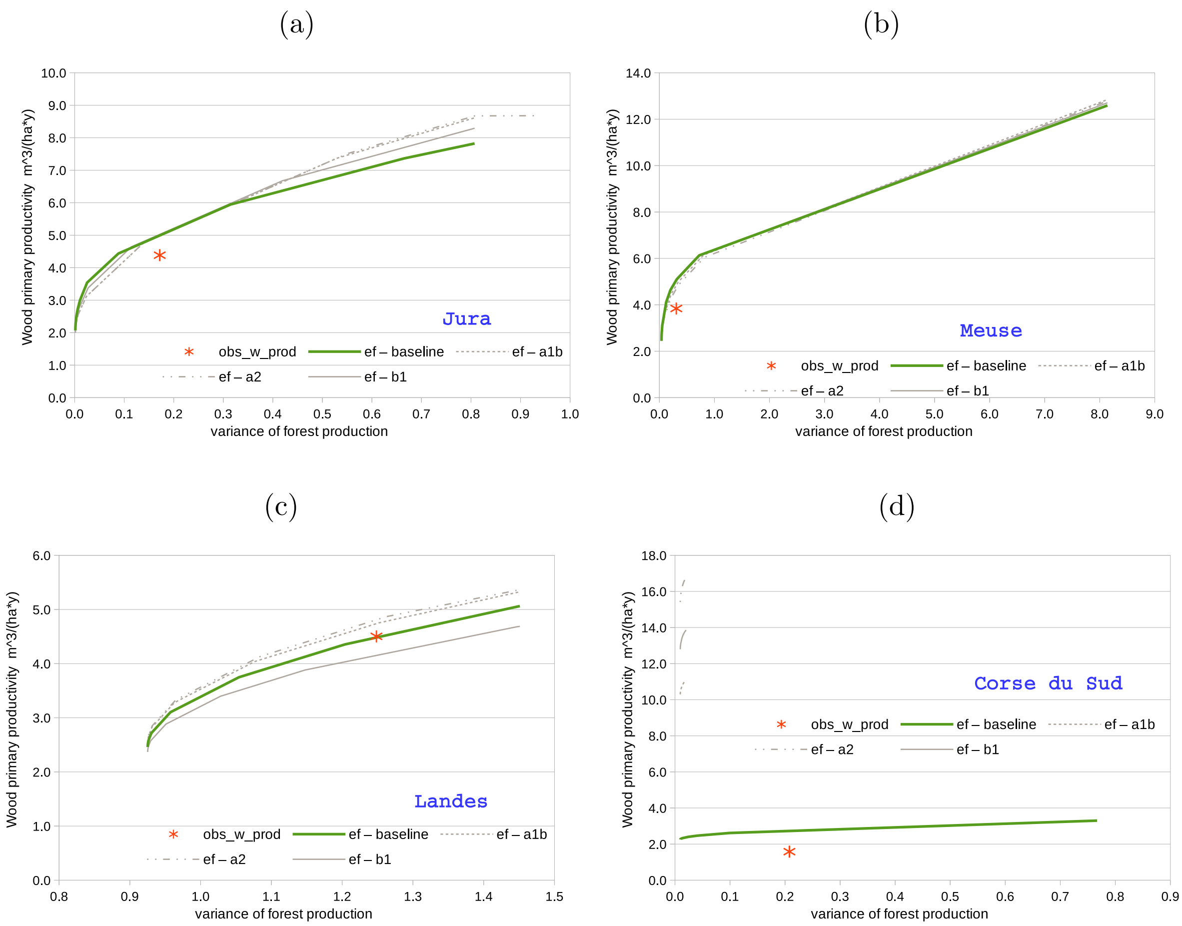
We note that the majority of departments display allocations similar to the national one, that is, close to the frontier with a preference for low variance at the expense of high productivity (the red point on the bottom left, which represents the actual species distribution, is close to the green line or EF). Among others, we can quote `Alpes Maritime`, `Jura` (Fig. 6a) and `Meuse` (Fig. 6b).

A few departments, distinguished by risk neutrality and high productivity (red star on the top right side), turn out to be on EF. This is the case of `Landes` (Fig. 6c), `Gironde` and `Haute Savoie`.

At last, some departments are distant from the optimal allocation (`Corse du sud` – Fig. 6d), `Pyrénées-Orientales` (the red star, on the bottom of the figure, is far from EF).

While the prevalence of private forest ownership and the presence of solid timber industry seem to favor risky allocations, as much as heterogeneous environmental conditions may constrain real forest allocations away from the optimal allocations, we do not have enough information to validate such assumptions.

Figure 6: Efficient frontiers and current allocations in four French departments



Efficient frontiers (curves) and actual allocation (red star) in four French departments. Baseline EF is 2009. Climate change scenarios are average values (2020-2100).

we intend to automatize it so as to quantitatively measure the risk aversion coefficient per department. At a constant risk-aversion, which is qualitatively provided for the moment, we could also measure the gap between EF and the observed allocation.

3.2 Maximizing for which objective?

As we switch objectives in the portfolio optimization, we obtain the performances, at the national level, such as described in Table 4, and the species allocations, such as depicted in Fig. 7. As the definition of risk aversion depends on the scale, the "intermediate" is defined for each objective with the risk aversion found at the national level (white dashed lines in Figs. 5 and 7).

While one would expect WP to be the highest when the objective is to maximize WP, as with CS and EV, it is proving to be true for risk neutrality only. As the risk aversion increases, risk counts more than productivity, such that optimizing for a cross objective may yield higher values.

When compared to Fig. 5, Fig. 7 shows that the carbon sequestration objective would roughly lead to the same portfolio. As previously stated, we observe differences only under the risk neutrality scenarios, where the high-dense wood species, like *Pseudotsuga menziesii* or *Fagus sylvatica*, supplant the low-dense wood species, like *Pinus sylvestris* and *Abies alba*.

The fact that, for the intermediate risk aversion, the empirical allocation is always between the *maxwood* and *maxvalue* objectives suggests that forest managers consider both objectives in real practice. For example, the observed wood production of $55.4 Mm^3 y^{-1}$ stands between the 52.1 EV objective and the 58.6 WP objective. Nevertheless, we cannot say, from the aggregate data, whether these two objectives are simultaneously accounted for during decision-making.

The empirical performance in carbon sequestration (78.8 Mt $CO_2 eq$ y^{-1}) is very close to what we find at the optimum and comparable to the French National Forestry Office figure[8], obtained when using the method of Loustau (2004) and Dupouey & Pignard (2001). Furthermore, the Kyoto protocol stipulates that, through forests, France ought to sequestrate around 66 Mt $CO_2 eq$ y^{-1} per year up to 2020 (Colin, 2014). This presumes that the Kyoto objectives are either too lax or that the French forests are highly efficient when comes to sequestrating. Thereby, should the France principally aim to produce wood, the Kyoto objective would be equal to the performance achieved in the baseline scenario at the full risk aversion.

In case the objective is to sequestrate carbon, the optimal species distribution is very similar to the one obtained when maximizing wood production.[9]

On the contrary, when the objective is to maximize the economic value of the wood production, considering the cost of waiting for the harvest, that is, the capital opportunity cost, yields very different species distributions. Indeed, forest producers cannot rapidly adapt to price variations.[10] At most, they can either decide to harvest the species highly-valued by the market at some point of time, or they can decide to postpone the harvesting until the prices increase. The second option is restricting in view of the rotation length, because over-anticipating harvesting would lead to a poor appreciation of the species set (as price increases with diameter), while over-postponing it would lead to the forest senescence with increased mortality risks. The relative unpredictability of prices makes the switch toward economic valorization very risky.

Optimizing for the economic valorization would also lead to a lower diversification, with a Shannon index of 1.79 (choosing e as base) for the observed risk aversion portfolio, against 2.09 when the objective corresponds to the timber or carbon maximization. In addition, under very high risk aversion, the species diversification is further reduced, with the Shannon index decreasing to 1.43. Such configuration may play a role on the effective risk borne by forest owners, since losses can be species specific (such as wood-boring insects and fungi), but lies outside the paper scope. In addition, such an option could be heavily detrimental on carbon sequestration and may radically change the landscape. This is rather unlikely, because many French forests are currently managed through the natural regeneration of existing species.

[8] Cf.:French National Forestry Office, key figures (in French)

[9] Coniferous store less carbon than broad leaves, but other discrepancies across species are limited.

[10] The same limits on the fixed covariance matrix is happening here.

Table 4: Optimization of wood production, carbon sequestration and economic valorization under three maximization objectives

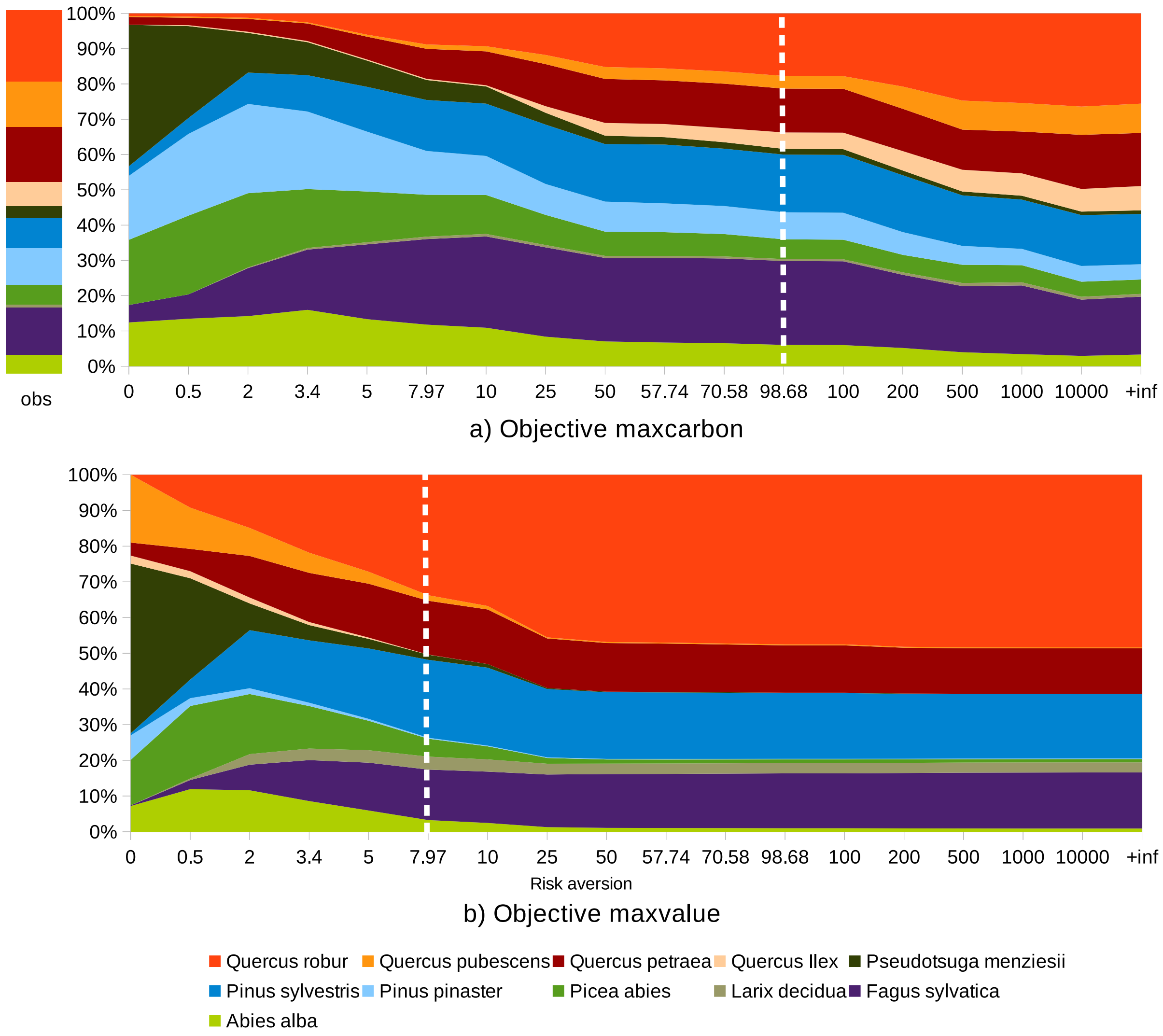
Climate change scenarios	a1b	a2	b1	baseline
Wood production ($M\ m^3\ y^{-1}$)				
(currently observed: 55.4 $M\ m^3\ y^{-1}$; 3.52 $Mc\ ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	45.6	47.0	46.5	43.8
- *obj maxcarbon*	49.6	51.1	50.3	47.7
- *obj maxvalue*	42.5	44.2	44.4	40.5
Intermediate risk aversion				
- *obj maxwood*	60.2	62.8	61.1	58.6
- *obj maxcarbon*	60.0	62.2	60.3	57.5
- *obj maxvalue*	55.1	57.1	55.4	52.1
Risk neutral				
- *obj maxwood*	136.7	139.5	137.0	136.3
- *obj maxcarbon*	134.1	136.8	133.6	132.7
- *obj maxvalue*	114.8	115.1	118.4	119.9
Carbon sequestration ($Mt\ CO_2eq\ y^{-1}$)				
(currently observed: 78.8 $Mt\ CO_2eq\ y^{-1}$; 5.01 $t\ CO_2eq\ ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	69.4	71.8	71.0	66.3
- *obj maxcarbon*	73.0	75.6	74.5	69.9
- *obj maxvalue*	66.1	69.0	69.1	62.6
Intermediate risk aversion				
- *obj maxwood*	84.8	88.6	86.7	82.0
- *obj maxcarbon*	85.0	88.5	86.4	81.4
- *obj maxvalue*	80.1	83.4	81.6	75.2
Risk neutrality				
- *obj maxwood*	169.9	173.1	172.9	170.4
- *obj maxcarbon*	173.2	176.2	176.3	174.4
- *obj maxvalue*	154.0	154.7	159.6	160.9
Economic valorization ($M€\ y^{-1}$)				
(currently observed: 3,789 $M€\ y^{-1}$; 240 € $ha^{-1}\ y^{-1}$)				
Full risk aversion				
- *obj maxwood*	3,849	3,894	3,880	3,745
- *obj maxcarbon*	3,707	3,754	3,735	3,604
- *obj maxvalue*	4,189	4,239	4,259	4,095
Intermediate risk aversion				
- *obj maxwood*	3,637	3,734	3,687	3,373
- *obj maxcarbon*	3,567	3,656	3,625	3,324
- *obj maxvalue*	4,506	4,618	4,543	4,131
Risk neutrality				
- *obj maxwood*	5,312	5,278	5,408	5,426
- *obj maxcarbon*	5,607	5,533	5,739	5,702
- *obj maxvalue*	5,623	5,566	5,767	5,841

4 Conclusions

Using the Markowitz mean-value (M-V) portfolio model, we studied forest planning that allows for considering the trade-off between productivity and risk in an explicit manner, focusing on the species allocation. When applied to the French metropolitan territory, our simulations yield a range of possibilities for forest managers to achieve three principal objectives which could be assigned to forest ecosystems.

Among the various services provided by forests, we mainly focused on wood production and carbon sequestration. In case of high level of risk aversion, we found that the empirical

Figure 7: Species allocation by risk aversion under three maximization objectives



performance is close to efficiency. Accordingly, at the current risk aversion level, changing the existing set-up would have no immediate foundation. In particular, given the small differences between the tree species in terms of carbon concentration, maximizing a portfolio for wood production amounts to maximizing it for carbon sequestration. This is our main result.

Knowing that carbon sequestration is almost fully correlated with productivity, when French authorities promote timber production, issued from sustainably managed forests, they turn out to target the fight against climate change. Nevertheless, would the Kyoto objectives, in terms of carbon sequestration, be more constraining in the future, the cursor should be moved to more risk neutrality, because the efficient frontier forecasts enhanced productivity at higher levels of risk. To do so, private mechanisms of risk sharing, such as the insurance contracts, should be implemented, especially in the regions, like the French South-West, where the forest owners are regularly subsidized in case of calamities. In other words, the portfolio approach highlights the constraints posed to climate change mitigation, where one's would increase carbon sequestration by moving through more risk neutrality, by the necessity of considering climate change adaptation and minimize the forest overall variance.

We also observe that forest managers both maximize physical wood production and soil expectation value. An interesting follow up could be to see whether this result is agent-dependent.

For example, we could differentiate between departments where the ownership is mostly public and those where the private ownership is preponderant.

While applying the methodology proposed in this study can help forest managers to choose between the forest investments, while balancing risk with productivity, it should not be approached in a strict normative sense.

First, spatial heterogeneity would likely lead to different combinations. Indeed, this method can only provide an "optimal" relative allocation of species, saying nothing about its spatial pattern. Coupling the Portfolio approach with spatially explicit models, like Macmillan 1992 did with the model by von Thünen (von Thünen, 1826), would allow to study the optimal species allocation with regard to the timber markets.

Second, modern forest management should consider a much broad number of dimensions that we could examine in numerical exercises, such as biodiversity, recreational value and landscape value. We observed that when we push for maximum production on top of increasing forest variance we also reduce diversification.

Third, this work focuses on species allocation as a way to manage risk, without considering forest management practices that can be simultaneously adopted to reduce risk, including changes in forest density and rotation length. In this aspect, one of the model limits is that it cannot capture other management activities than the changes in species compositions. All the measured wood increment is a gain in productivity. In the economic valorization perspective, it is assumed to be entirely available for the final harvest at the end of the rotation period, thus disregarding thinning and mortality which ought to reduce the final standing timber volume. This is particularly true with the long-standing broadleaved trees, where the difference between the standing volumes – computed by multiplying the annual productivity by the rotational period – and the actual volumes measured in mature forests is greater than with the coniferous trees subject to short-term rotation. Nevertheless, the lack of regeneration and maintenance costs in the model favors the short-term rotation species.

Finally, it is still unknown how climate change would not only affect the average productivity, but also the structure of the variance/covariance matrix, which could lead to different "optimal" portfolios. The computed portfolios indeed consider the variance of production (and, in the economic valorization objective, of prices) of the historical data and are "optimal" in relation to the observed aleatory disturbances. Still, a portfolio that is highly resilient to the current climate is the most recommendable one for the unknown future.

Nevertheless, in order to provide foresters with wide and balanced tools for the long-term forest management, we believe that the results obtained from this type of studies could partner with those derived from more traditional approaches.

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A Simulator source code, input data and complete output

The supplementary material is composed of 2 parts:

A.0.1 Simulation program

`loop.7z` includes the program used to produce the simulations reported in this paper (a python script) and the required data. Unfortunately two input files, productivities and climate change multipliers, are not publicly available as we do not hold their copyright.

A.0.2 Complete output data

`output.7z` contains two OpenDocument spreadsheet files. `spAllocation.ods` contains, for each run simulation, the portfolio's weight for each species. `depPerformances.ods` includes instead the consequent "performances" of such optimal portfolios for the dimensions analysed in the text.